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Limited-Angle and Few-View Image Reconstruction Using Point Spread Functions

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ABSTRACT

Limited-angle tomography and few-view tomography are extremely ill-conditioned problems. A usual way to reconstruct an image in limited-angle tomography or in few-view tomography is via an iterative algorithm that minimizes a two-term objective function. The objective function consists of a data fidelity term and a Bayesian term. A common Bayesian term is the total variation (TV) norm of the image. This paper proposes a different way to reconstruct the image in limited-angle tomography and few-view tomography by iteratively deconvolving the point spread functions (PSFs). Two PSFs are considered: the PSF of the filtered backprojection (FBP) and the PSF of pure backprojection. The method of using the PSF of pure backprojection gives better results for both limited-angle tomography and few-view tomography studies.

Keywords

Limited-angle tomography, Few-view tomography, Total variation prior, Point spread function, Deconvolution.

Introduction

Limited-angle tomography uses projections acquired in an angular range smaller than the full-scan angular range [1-4]. In some applications, the scanning angular range is restricted by system hardware setup. For example, in two-dimensional (2D) parallel-beam imaging geometry, the full-scan angular range is 180°. For 2D fan-beam imaging geometry, a complete scan requires 180° plus the fan angle [5]. For the three-dimensional (3D) cone-beam imaging geometry, none of the 2D trajectories are sufficient to provide a full set of the projections. A none-planar trajectory that satisfies Tuy's condition is required to measure a complete data set [6]. When projection data is incomplete, typical artifacts in the reconstructed images are the image blurring in the directions where the projections are not available [7]. When a full-scan data acquisition is not possible, we must develop special algorithms to reconstruct the images.

Few-view tomography uses much fewer projection views in data acquisition than requested by the Nyquist sampling criterion [2,8,9]. If the image array is 512 × 512, the usual x-ray computed tomography (CT) scan has around 1000 to 2000 projection views. For many applications, we must acquire projection data only at a small number of views, and the Nyquist sampling criterion is violated. One example of such situations is radiation therapy monitoring. When the projection views are not sufficient, angular aliasing artifacts may appear in the reconstruction. The artifacts appear as streaking tangent lines, passing through the high-contrast edges. No standard and well-developed image reconstruction algorithms are available if the projection views are extremely low. Both limited-angle tomography and few-view tomography have vast important applications in healthcare and industrial non-destructive testing and inspection. Due to the severe ill-condition, the image reconstruction problem for limited-angle tomography and few-view tomography is still open and is far from solved.

The current image reconstruction technique for limited-angle tomography and few-view tomography is iterative algorithms that minimize objective functions with a data-fidelity term and some Bayesian terms. A common Bayesian term is the TV norm of the image [10]. The data-fidelity term checks if the forward projections of the estimated image match the acquired projections. The Bayesian terms enforce some desired properties in the reconstruction. For example, some desired properties are smoothness of the image and the piecewise-constant structure.

This paper introduces an alternative technique that iteratively deconvolves the point spread function (PSF) of the raw backprojection, which may or may not use ramp filtering before backprojection.

Methods

If we construct the image with the filtered backprojection (FBP) algorithm, the reconstruction procedure is linear and shift invariant [11]. Therefore, the point spread function (PSF), h_{FBP}, of the FBP algorithm can be used to characterize the backprojection result for limited-angle and few-view situations. We denote the FBP reconstruction as f_{FBP} and the true image as f. The two-dimensional (2D) convolution relationship is given as

$$f_{FBP} = f * h_{FBP}. \quad (1)$$

Without the application of the ramp filter, the pure backprojection of the projection measurements yields an image b. This pure backprojection procedure is also linear and shift invariant. Thus, the PSF, h_{back}, of pure backprojection (without using the ramp filter) can be used to characterize the backprojection results for limited-angle and few-view situations. The 2D convolution relationship for these backprojection situations is given as

$$b = f * h_{back}. \quad (2)$$

In limited-angle tomography, the scanning range is assumed to be $[0,\varphi]$. The PSF for the FBP backprojection is

$$h_{FBP}(r,\theta) = \int_0^{\varphi} g_{ramp}(s)|_{s=rcos(\theta-\phi)} d\phi\,, \quad (3)$$

where $g_{ramp}(s)$ is the 1D ramp filter in the spatial domain, and its Fourier domain expression is $|\omega|$, where ω is the frequency. In the special case of $\varphi=\pi$, the PSF $h_{FBP}(r,\theta)$ is approximately a Dirac delta function $\delta(r)$, and the FBP algorithm gives an almost perfect image from noiseless projections.

The PSF for pure backprojection is

$$h_{back}(r,\theta) = \int_0^{\varphi} \delta(s)|_{s=rcos(\theta-\phi)} d\phi = \int_0^{\varphi} \delta\big(rcos(\theta-\phi)\big) d\phi = \frac{1}{r}\int_0^{\varphi} \delta\big(cos(\theta-\phi)\big) d\phi. \quad (4)$$

This expression (4) implies that the PSF $h_{back}(r,\theta)$ behaves essentially as $1/r$ multiplied with an angular function. This angular function has a wedge shape and its value is 1 in the scanned angular region and is 0 in the unscanned angular region.

For few-view tomography, it is proper to replace the integrals in (3) and (4) by summations because $d\phi$ is no longer small. Therefore, (3) and (4) become (5) and (6), respectively, as

$$h_{FBP}(r,\theta) = \frac{\pi}{N}\sum_{k=1}^{N} g_{ramp}(s)|_{s=rcos(\theta-\phi_k)}\,, \quad (5)$$

$$h_{back}(r,\theta) = \frac{\pi}{N}\sum_{k=1}^{N} \delta(s)|_{s=rcos(\theta-\phi_k)}\,, \quad (6)$$

where ϕ_k is a view angle, k=1,2,...,N. Here, N is the total number of views.

In our computer simulations, the PSFs were numerically calculated by backprojecting a point source at the origin: $h_{FBP}(r,\theta)$ with a ramp filter and $h_{back}(r,\theta)$ without a ramp filter before backprojection of $\delta(s)$, where s is the coordinate on the detector.

The objective function for the image reconstruction tasks was

$$Objective\ Function(f) = \|h * f - \hat{f}\|_{L_2}^2 + \lambda \times TV(f), \quad (7)$$

where f is to be reconstructed image, $TV(f)$ is the TV norm of image f, and λ is a user specified parameter. A gradient descent algorithm was used to minimize the objective function (7). The 2D convolution $h*f$ is performed twice in each iteration [12]. Since the PSF h is a large array, it is computationally more efficient to perform convolution in the 2D Fourier domain as matrix multiplication than in the spatial domain as 2D convolution.

We denote

Method 1: f is the filtered backprojection (FBP) and h=h_{FBP}.

Method 2: f is the pure backprojection and h=h_{back}.

Results

Matlab was used to implement the proposed deblurring algorithms. The image size was 256 × 256. The Shepp-Logan phantom was used. The imaging geometry was 2D parallel beam. In the limited-angle tomography studies, the scanning angle was 135° with 180 views. In the few-view tomography studies, the number of views was 18 over 180°. A gradient descent algorithm was used to minimize the objective function (7). Some user-defined parameters in the gradient descent algorithm are given in Table 1.

Table 1: Some parameters used in the gradient descent algorithm.

Algorithm	Step size	λ	Number of iterations
Limited Angle, Method 1	1.2	0.0007	200,000
Limited Angle, Method 2	15.2	0.0007	200,000
Few view, Method 1	0.01	0.0007	1,000
Few view, Method 2	1.0	0.0004	50,000

Four studies are reported in Figure. 1~4. Figures 1 and 2 are the results for limited-angle tomography, with the scanning angle of 135°. Figures 3 and 4 are the results for few-view tomography, with 18 views over the scanning angle of 180°. Figures 1 and 3 are the results of Method 1, while Figures 2 and 4 are the results of Method 2.

Both methods can improve the image shapes in limited-angle tomography and can remove the streaking lines in the few-view tomography. It is observed that Method 2 is better than Method 1, in the sense that the final reconstructions of Method 2 have less artifacts.

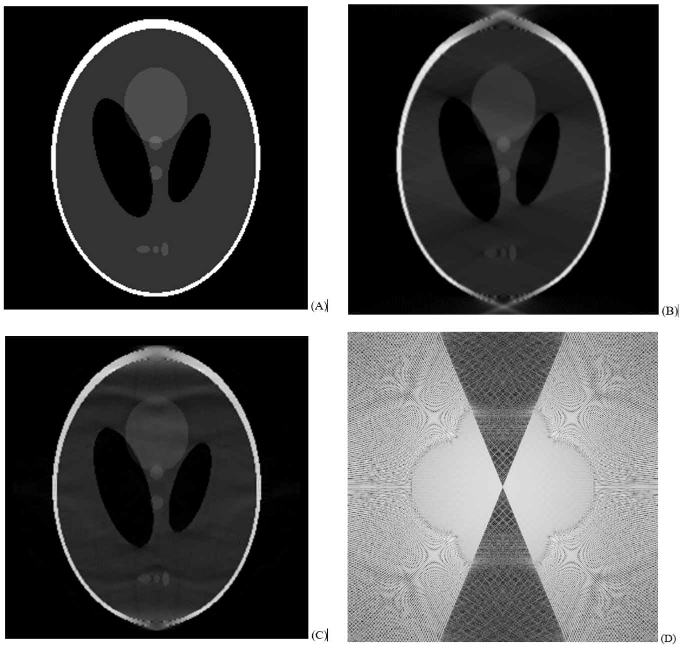

Figure 1: Limited-angle Method 1 study. (A) Shepp-Logan phantom; (B) Raw FBP reconstruction; (C) Final FBP reconstruction; (D) PSF h_{FBP} in log scale.

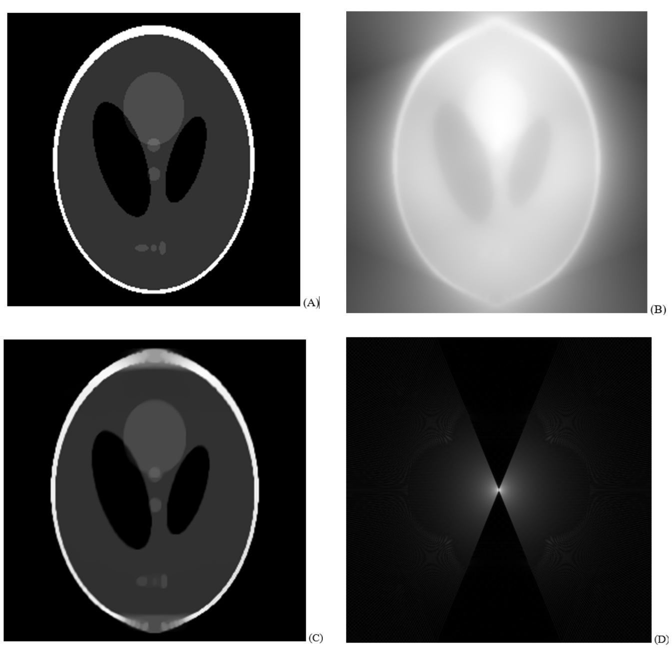

Figure 2: Limited-angle Method 2 study. (A) Shepp-Logan phantom; (B) Pure backprojected image; (C) Final deblurred reconstruction starting from pure backprojection; (D) PSF h_{back} in log scale.

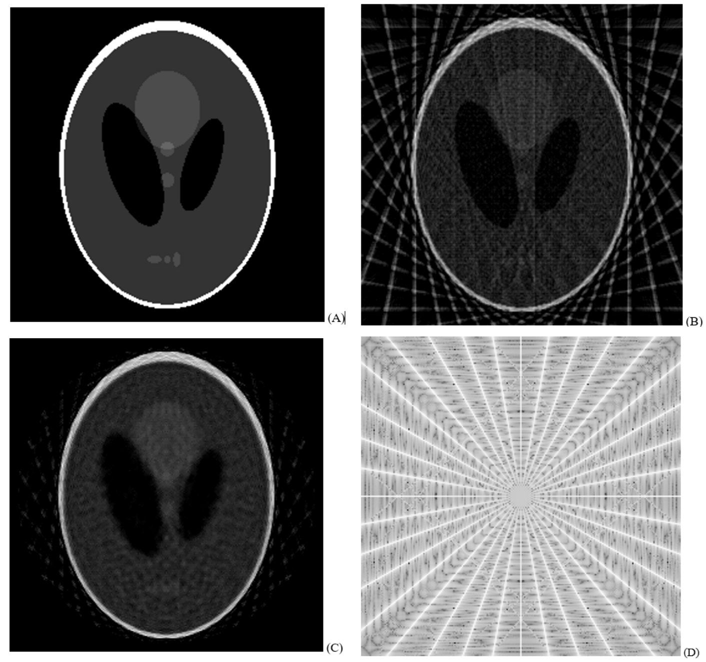

Figure 3: Few-view Method 1 study. (A) Shepp-Logan phantom; (B) Pure backprojected image; (C) Final deblurred reconstruction starting from pure backprojection; (D) PSF h_{back} in log scale.

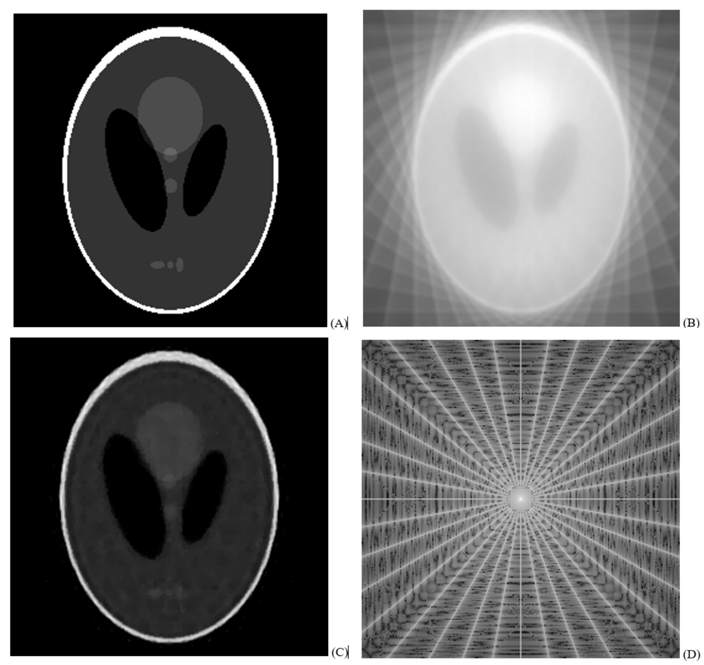

Figure 4: Few-view Method 2 study. (Upper Left) Shepp-Logan phantom; (Upper Right) Pure backprojected image; (Lower Left) Final deblurred reconstruction starting from pure backprojection; (Lower Right): PSF h_{back} in log scale.

Conclusion

Limited-angle tomography and few-view tomography are severely ill-conditioned, and their stable solutions are yet to be sought. Different methods may have different performances. For example, the closed-form Weiner filter method is not as stable as iterative deconvolution. This paper presented two deblurring methods for image reconstruction. The first method uses the PSF of ramp filtered backprojection. The second method uses the PSF of pure backprojection.

Computer simulations show that the streaking line artifacts are successfully removed by proposed Method 1 and Method 2. The simulations also show that the second method outperforms the first method in terms of artifact severeness in both limited-angle studies and few-view studies. In fact, the artifacts in the final reconstruction (see image C in the figures) also somehow already exist in the backprojected image (see image B in the figures). This may indicate that the ramp filter enhances the artifacts.

No firm conclusions can be drawn by using a small number of computer simulations. More intensive investigations are needed in future studies.

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